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Three-photon Decay of Neutral Pions[1)]

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In connection with the possibility of C violation in strong or electromagnetic interactions, we estimate the probabilities for the decays $\pi^0 \to 3\gamma$ and $\pi^0 \to 4\gamma$.

The process $\pi^0 \to 3\gamma$, which may proceed only if either the strong or electromagnetic interactions violate C parity, has been recently discussed in a number of papers,[1–3] particularly in connection with possible violation of CP.

The angular distributions, as well as general properties of the amplitude, have been studied in most detail in the paper of Berends,[4] who starts from the simplest possible effective Hamiltonian violating $T(CP)$. He also estimates the quantity $R_{32} = w(\pi^0 \to 3\gamma)/w(\pi^0 \to 2\gamma)$, by taking the ratio of the phase space for the two processes normalized in a specified way and by introducing a factor which is supposed to represent suppression of the C-violating interaction in comparison with the conventional one. The value $R_{32} = 10^{-6}$ obtained by him, seems to us somewhat overestimated.

A more correct value for this quantity was obtained by Grishin and Kopylov.[5] They calculated the lifetime of hypothetical light mesons which can decay into three or four photons, making use of "simplest local effective matrix element" (or, what is the same, local Lagrangian). Their results can be directly generalized to the case of the decay $\pi^0 \to 3\gamma$. At that their matrix element for the process $0^{+-} \to 3\gamma$ with conservation of C and P may also describe the decay $0^{-+}(\pi^0) \to 3\gamma$ with violation of C and P (CP conserved). Their process $0^{--} \to 3\gamma$ corresponds in our terminology to the decay $\pi^0 \to 3\gamma$ with violation of C and conservation of P. However, the matrix element for the process $0^{--} \to 3\gamma$ used by these authors vanishes under symmetrization with respect to the photon variables.

Thus the upper bound for the probability of the three-photon decay may be smaller than the quantity 10^{-9} sec^{-1} obtained with the help of this matrix element. Moreover, the magnitude of the probability for the four-photon decay of the vector particle, calculated in that paper, cannot be used to estimate the probability of the four-photon decay of the π^0, since calculation shows that these quantities differ by approximately three orders of magnitude.[2)] Below we give estimates of the probabilities of three- and four-photon decay of π^0.

The invariant amplitude A_3 of the three-photon decay for both possibilities $CPA_3 = A_3$ (conservation of CP) and $CPA'_3 = -A'_3$ (violation of CP) is determined by the requirements of Lorentz invariance and gauge invariance to within two unknown functions, and may be represented in the form

$$A_3 = [1 + P(13) + P(23)] \times \{(t-u) f_{\rho\lambda}^1 f_{\rho\lambda}^2 f_{\mu\nu}^3 k_\mu^1 k_\nu^2 a(s,t,u)\} + (s-t)(t-u)(u-s) f_{\mu\nu}^1 f_{\nu\rho}^2 f_{\rho\mu}^3 b(s,t,u),$$

$$A_3' = [1 + P(13) + P(23)] \times \{(t-u) \tilde{f}_{\rho\lambda}^1 f_{\rho\lambda}^2 f_{\mu\nu}^3 k_\mu^1 k_\nu^2 c(s,t,u) + (s-t)(t-u)(u-s) \tilde{f}_{\mu\nu}^1 f_{\nu\rho}^2 f_{\rho\mu}^3 d(s,t,u)\},$$

where the functions a and c are symmetric with respect to the last two variables, and b and d are symmetric with respect to all variables. Here $f_{\alpha\beta}^i$ is the electromagnetic tensor in the momentum representation, describing the i-th photon, and $\tilde{f}_{\alpha\beta}^i = \frac{1}{2}\epsilon_{\alpha\beta\gamma\sigma} f_{\gamma\sigma}^i$ is the tensor dual to it; $s = k_1 \cdot k_2$, $t = k_1 \cdot k_3$, $u = k_2 \cdot k_3$ are scalar products of the corresponding four-momenta. From dimensional considerations it follows that

$$a \sim c \sim M_3^{-7}, \qquad b \sim d \sim M_3^{-9},$$

where M_3 is a parameter with the dimension of mass, characterizing the three-photon decay. It is clear that only the masses of the charged hadrons, into which the π^0 dissociates virtually, can serve as this parameter. Therefore $M_3 \gtrsim m = m_\pi$.

Taking into account the smallness of the phase space, one may neglect the functional dependence of the quantities a and c and omit b and d. The calculation shows that the contribution of the omitted terms to the probability is of the order m^2/M_3^2 relative to the included terms, and therefore cannot substantially change its order of magnitude. Thus let us set

$$a = e^3\eta_3 M_3^{-7}, \quad c = e^3\eta_3' M_3^{-7}, \quad b = d = 0,$$

where e is the elementary charge and η is a dimensionless parameter. The invariant matrix element A_3 obtained in this approximation coincides with the corre-

[1)]Reported at the Sixth All-union Conference on the Theory of Elementary Particles, Uzhgorod, October, 1965.

[2)]The author is grateful to V. G. Grishin and G. I. Kopylov for discussion of these points.

sponding expression in [5] (process $0^{+-} \to 3\gamma$), and A'_3 with the expression in [4].

Having in mind a further comparison of the probability of the three-photon decay of the π^0 with the probability of two and four-photon decays, let us assign to these processes the simplest invariant amplitudes

$$A_2 = \frac{e^2\eta_2}{M_2}\tilde{f}_{\alpha\beta}{}^1 f_{\alpha\beta}{}^2,$$

$$A_4 = \sum_p \frac{e^4}{M_4{}^5}[\eta_4 f_{\alpha\beta}{}^1 f_{\alpha\beta}{}^2 f_{\gamma\delta}{}^3 f_{\gamma\delta}{}^4 + \xi_4 f_{\alpha\beta}{}^1 f_{\beta\gamma}{}^2 f_{\gamma\delta}{}^3 f_{\delta\alpha}{}^4],$$

$$A_4' = \sum_p \frac{e^4}{M_4{}^5}[\eta_4' \tilde{f}_{\alpha\beta}{}^1 f_{\alpha\beta}{}^2 f_{\gamma\delta}{}^3 f_{\gamma\delta}{}^4 + \xi_4' \tilde{f}_{\alpha\beta}{}^1 f_{\beta\gamma}{}^2 f_{\gamma\delta}{}^3 f_{\delta\alpha}{}^4].$$

Estimates of the parameters M, η and ξ will be given below with the help of certain diagrams.

By carrying out standard calculations we obtain (n is the number of photons)

$$w_n = \frac{1}{(2\pi)^{3n-4} n! \cdot 2m} \times \int \prod_{i=1}^{n} \frac{d^3k^i}{2\omega^i} \delta^4\left(p_\pi - \sum_{i=1}^{n} k^i\right) |A_n|^2,$$

$$w_2 = \alpha^2\eta_2{}^2\pi m \left(\frac{m}{M_2}\right)^2,$$

$$\frac{w_3}{\eta_3^2} = \frac{w_3'}{\eta_3'^2} = \alpha^3 m \left(\frac{m}{M_3}\right)^{14} \frac{1}{9! \cdot 2^{11}},$$

$$w_4 = \frac{\alpha^4}{\pi} m \left(\frac{m}{M_4}\right)^{10} \frac{(8\eta_4^2 + 7\eta_4\xi_4 + 2\xi_4^2)}{80 \cdot 9!},$$

$$w_4' = \frac{\alpha^4}{\pi} m \left(\frac{m}{M_4}\right)^{10} \frac{16\eta_4'^2 + 8\eta_4'\xi_4' + \xi_4'^2}{256 \cdot 9!},$$

whence

$$R_{32} = \frac{w_3}{w_2} = \left(\frac{\eta_3}{\eta_2}\right)^2 \frac{\alpha}{\pi} \frac{1}{2^{11} \cdot 9!} \left(\frac{m}{M_3}\right)^{14} \left(\frac{M_2}{m}\right)^2,$$

$$R_{43} = \frac{w_4}{w_3} = \frac{\alpha}{\pi} \frac{128(8\eta_4^2 + 7\eta_4\xi_4 + 2\xi_4^2)}{5\eta_3^2},$$

$$R'_{43} = \frac{w_4'}{w_3} = \frac{\alpha}{\pi} \frac{8(16\eta_4'^2 + 8\eta_4'\xi_4' + \xi_4'^2)}{\eta_3^2}.$$

Let us pass now to an estimate of the parameters M, η, and ξ. As was shown by Schwinger,[6] the two-photon decay of the pion is quite well described by

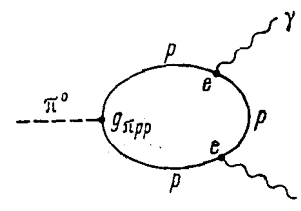

Fig. 1.

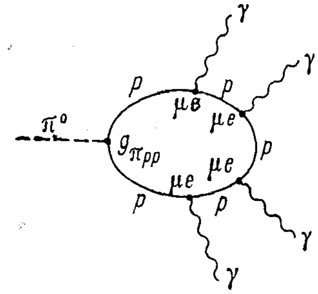

Fig. 2.

the simple diagram (Fig. 1) with the proton-antiproton pair in the intermediate state. In that case

$$M_2 = M_p, \quad \eta_2 = g_{\pi pp}/8\pi^2.$$

Calculation of the analogous diagram (Fig. 2) in the four-photon case involves technical difficulties. To simplify the calculation, the Dirac interaction $e\gamma^\mu A_\mu$ in the electromagnetic vertices was replaced by the Pauli interaction $(\mu e/8M)\sigma_{\mu\nu}F_{\mu\nu}$. In this model

$$M_4 = M_p, \quad \eta_4 = -\frac{9\mu^4}{8\pi^2}\frac{1}{128},$$

$$\eta_4' = 5\eta_4, \quad \xi_4 = 32\eta_4, \quad \xi_4' = 48\eta_4.$$

In the three-photon case the analogous diagram (Fig. 3) with spinor particles in the intermediate state gives a nonzero result only if the masses are different, i.e., magnetic transitions of the type $\Sigma \to \Lambda\gamma$ play a role. In that case the violation of C may occur either in the strong or in the electromagnetic vertex. In this model

$$M_3 = \frac{M_\Sigma + M_\Lambda}{2},$$

$$\eta_3 = \frac{g_{\pi\Sigma\Lambda}}{8\pi^2}\frac{\mu^3(M_\Sigma - M_\Lambda)}{1260}, \quad \eta_3' = 2\eta_3.$$

If the intermediate particles are vector particles, then, by taking as the C-violating mechanism the current given by Lee[7]

$$K_\mu = ie\beta\frac{\partial}{\partial x_\mu}(\bar{\omega}_\mu{}^0\Phi_\nu - \bar{\omega}_\nu{}^0\Phi_\mu)$$

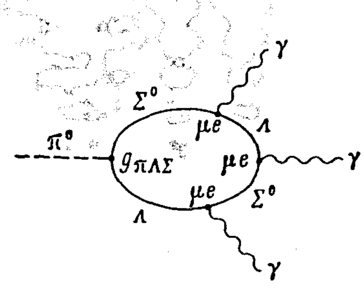

Fig. 3

and retaining the main terms in the estimates, we obtain

$$M_3 = M_{\omega(\phi)}, \quad \eta_3 = \frac{1}{8\pi^2}\frac{\lambda_{\pi\omega\phi}}{M_3}\frac{\beta^3}{24};$$

where $\lambda_{\pi\omega\phi}$ is the $\pi\omega\phi$ coupling constant, with dimension of mass; β is a dimensionless constant characterizing the intensity of the C-violating interaction relative to the conventional one. The calculations of [5] correspond to $M = m$, $\eta_n = (4\pi)^{-n/2}$.

From these estimates it is seen that the factor

$$\left(\frac{\eta_3}{\eta_2}\right)^2 \left(\frac{m}{M_3}\right)^{14} \left(\frac{M_2}{m}\right)^2$$

can be considered as smaller than unity, even without the additional hypothesis that the C-violating interaction is weak, and then

$$R_{23} \leqslant \frac{\alpha}{2\pi}\cdot 10^{-6} \approx 10^{-9}.$$

It follows from here that the process under consideration cannot be used as a good indicator for conservation of C-parity in strong and electromagnetic interactions, since it is suppressed for kinematical reasons.

As to the quantities R_{43} and R'_{43}, it is not possible to assert from these estimates that they are certainly small. For certain reasonable values of the parameters M, η, and ξ they may be comparable with unity and even substantially exceed it. It is therefore of interest to estimate the decay probability $w_{4\omega_0}$ when the energy of one of the photons is below the threshold of resolution of the apparatus ω_0, i.e., the probability of imitating three-photon decays. Omitting terms of higher order in ω_0/m we obtain

$$\frac{w_{4\omega_0}}{w_4} = 140\left(\frac{2\omega_0}{m}\right)^4$$

regardless of the form of the interaction. For values of ω_0 equal to $m/20$ and $m/10$ we obtain for $w_{4\omega_0}/w_4$ the values 7/500 and 28/125, respectively. Taking into account the comments regarding R_{43} we see that the imitation of the three-photon decay may be substantial.

In order to distinguish the true three-photon decay from its imitation it is necessary to compare their energy distributions or, which is the same, their angular distributions. For a true three-photon decay the energy distribution in the rest frame of the π^0 has the form

$$dw_3 \sim (m-2\omega_1)(m-2\omega_2)(m-2\omega_3)$$
$$\times[(\omega_1-\omega_2)^2+(\omega_1-\omega_3)^2+(\omega_2-\omega_3)^2]\,d\omega_1 d\omega_2,$$
$$\omega_1+\omega_2+\omega_3 = m.$$

For the imitation

$$dw_{4\omega_0} \sim a[\omega_1^2(m-2\omega_1)^2+\omega_2^2(m-2\omega_2)^2$$
$$+\omega_3^2(m-2\omega_3)^2] - b[\omega_1\omega_2(m-2\omega_1)(m-2\omega_2)$$
$$+(\omega_1\omega_3)(m-2\omega_1)(m-2\omega_3)$$
$$+\omega_2\omega_3(m-2\omega_2)(m-2\omega_3)$$
$$-{}^3/_8(m-2\omega_1)(m-2\omega_2)(m-2\omega_3)],$$

where

$$a = 80\eta_4^2 + 56\eta_4\xi_4 + 13\xi_4^2,$$

$$b = 4(16\eta_4^2 + 8\eta_4\xi_4 + \xi_4^2)$$

$$a = b = 4(16\eta_4'^2 + 8\eta_4'\xi_4' + \xi_4'^2).$$

The characteristic feature of the energy distributions of the true three-photon decay is the vanishing of the probability for the appearance of three-photons in the symmetric configuration $\omega_1 = \omega_2 = \omega_3$; with that, as can be seen from the general form of the amplitudes A_3 and A'_3, this property does not depend on the approximations used.

The four-photon decay (in the chosen model) does not possess this property for any value of η and ξ.

The angular distributions are obtained from the energy distributions by the substitution

$$\omega_1 = \frac{m\sin\theta_{23}}{\sin\theta_{12}+\sin\theta_{13}+\sin\theta_{23}},$$

$$\omega_2 = \frac{m\sin\theta_{13}}{\sin\theta_{12}+\sin\theta_{13}+\sin\theta_{23}},$$

$$\omega_3 = \frac{m\sin\theta_{12}}{\sin\theta_{12}+\sin\theta_{13}+\sin\theta_{23}},$$

$$d\omega_1 d\omega_2 = \frac{D(\omega_1\omega_2)}{D(\theta_{13},\theta_{23})}d\theta_{13}d\theta_{23}.$$

Here θ_{ik} is the angle between the direction of emission of the i-th and k-th photon in the rest frame of π^0,

$$\theta_{12}+\theta_{13}+\theta_{23} = 2\pi.$$

The author is grateful to V. M. Kut'in, L. I. Lapidus and V. I. Petrukhin for numerous useful discussions of questions relevant to this paper.

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